COMPENSATING FOR HEAT FLOWS ALONG CURRENT LEADS

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UDC 536.2.083

The possibility of compensating for heat flows along current leads by selection of the geometry, material, and operating mode of the conductors is considered.

The problem of compensating for thermal losses along current-carrying conductors arises in the design of measurement and monitoring circuits using sensors operating in the thermoanemometer mode (hot-wire anemometers, vacuum gauges, conductometric gas analyzers, etc.). A similar problem occurs in radio electronics when heat-sensitive components must be thermally insulated from hotter or cooler structural elements; it is also important in cryogenics in the design of economical current leads. Special heat-compensation arrangements are sometimes used under such conditions [1].

Here we propose a method of compensating for the heat flow over current-carrying leads; it is based on selection of geometry, material, and operating regime for such leads. The differential equation characterizing the steady-state distribution of heat flux in a conductor with current has the form

$$\lambda s \frac{d^2\vartheta}{dx^2} - \alpha F\vartheta + \frac{I^2\rho(\vartheta)}{s} = 0. \tag{1}$$

The $\rho(\vartheta)$ relationship is taken to be linear,

$$\rho(\vartheta) = \rho_0(1 + \beta\vartheta). \tag{2}$$

The thermal conductivity λ of the given conductor, whose ends are at temperatures ϑ_0 and ϑ_1, is assumed to be constant:

$$\lambda = \frac{1}{\vartheta_1 - \vartheta_0}\int_{\vartheta_0}^{\vartheta_1} \lambda(\vartheta)\, d\vartheta. \tag{3}$$

When we allow for (2) and (3) and the boundary conditions $\vartheta(0) = \vartheta_0$ and $\vartheta(l) = \vartheta_1$, Eq. (1) has a solution in three forms:

$$\vartheta(x) = \frac{\left(\vartheta_0 + \frac{c}{k^2}\right)\sin k(l-x) + \left(\vartheta_1 + \frac{c}{k^2}\right)\sin kx}{\sin kl} - \frac{c}{k^2}, \tag{4A}$$

where

$$c = \frac{I^2\rho_0}{\lambda s^2}, \quad k^2 = \frac{I^2\beta\rho_0 - \alpha Fs}{\lambda s^2}, \quad I > \sqrt{\frac{\alpha Fs}{\beta\rho_0}};$$

$$\vartheta(x) = \frac{\left(\vartheta_0 - \frac{c}{k^2}\right)\operatorname{sh} k(l-x) + \left(\vartheta_1 - \frac{c}{k^2}\right)\operatorname{sh} kx}{\operatorname{sh} kl}, \tag{4B}$$

and c is defined above, $k^2 = \alpha Fs - I^2\beta\rho_0/\lambda s^2$, $I < \sqrt{(\alpha Fs/\beta\rho_0)}$;

$$\vartheta(x) = \vartheta_0 + (\vartheta_1 - \vartheta_0)\frac{x}{l} + \frac{cx}{2}(l - x), \tag{4C}$$

where c has been defined above, $I = \sqrt{(\alpha Fs/\beta\rho_0)}$, $k = 0$.

ITMO, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 27, No. 2, pp. 277-281, August, 1974.

Case "A" may be characterized as the mode in which there is intense heating of the conductor by electric current, and the additional Joule losses owing to the variation of conductor resistance with temperature exceed the heat transfer between the surface of the conductor and the environment. In case "B" heat transfer from the surface exceeds the additional Joule losses. Case "C" may be treated as a thermal compensation regime in which the additional Joule losses are balanced by heat transfer between the surface and the environment. The conductor is cooled, as it were, solely by heat conduction through its material with constant release of heat due to heating by the current.

The heat-flux expressions corresponding to each of the cases above are

$$q(x) = \lambda sk \frac{\left(\vartheta_0 - \frac{c}{k^2}\right)\cos k(l-x) - \left(\vartheta_1 - \frac{c}{k^2}\right)\cos kx}{\sin kl}, \tag{5A}$$

$$q(x) = \lambda sk \frac{\left(\vartheta_0 - \frac{c}{k^2}\right)\operatorname{ch} k(l-x) - \left(\vartheta_1 - \frac{c}{k^2}\right)\operatorname{ch} kx}{\operatorname{sh} kl}, \tag{5B}$$

$$q(x) = \lambda s\left[(\vartheta_0 - \vartheta_1)\frac{1}{l} - c\left(\frac{l}{2} - x\right)\right]. \tag{5C}$$

Setting $q(0)$ and $q(l)$ to zero, we can obtain the conditions for compensation of the heat flux at points $x = 0$ and $x = l$ from these equations.

Equation (5A) has the following solutions:

$$\text{for } x=0 \quad l = \frac{1}{k}\arccos\frac{\vartheta_1 - \frac{c}{k^2}}{\vartheta_0 - \frac{c}{k^2}}, \quad \vartheta_1 < \vartheta_0, \tag{6A}$$

$$\text{for } x=l \quad l = \frac{1}{k}\arccos\frac{\vartheta_0 - \frac{c}{k^2}}{\vartheta_1 - \frac{c}{k^2}}, \quad \vartheta_0 < \vartheta_1. \tag{7A}$$

From (6A) and (7A) we can find the relationships between k and l that will ensure absence of heat in flux along a conductor to points $x = 0$ or $x = l$; the same equations also show that for a conductor operating in mode "A" the heat flux can only be compensated at the hotter end. By analogy, for a conductor operating in mode "B"

$$\text{for } x=0 \quad l = \frac{1}{k}\operatorname{Arch}\frac{\vartheta_1 - \frac{c}{k^2}}{\vartheta_0 - \frac{c}{k^2}}, \tag{6B}$$

$$\text{for } x=l \quad l = \frac{1}{k}\operatorname{Arch}\frac{\vartheta_0 - \frac{c}{k^2}}{\vartheta_1 - \frac{c}{k^2}}. \tag{7B}$$

The Arch x function is meaningful for arguments exceeding unity; it therefore follows from (6B) and (7B) that for a conductor operating in mode "B" the heat flux can only be compensated for the following conductor regimes:

$$\text{for } x=0 \quad \vartheta_1 > \vartheta_0 > \frac{c}{k^2}, \quad \vartheta_1 < \vartheta_0 < \frac{c}{k^2};$$

$$\text{for } x=l \quad \vartheta_0 > \vartheta_1 > \frac{c}{k^2}, \quad \vartheta_0 < \vartheta_1 < \frac{c}{k^2}.$$

When the numerous restrictions imposed prevent compensation of the heat flux in a conductor we can use (5A) and (5C) to compute the influxes of heat.

For a conductor in mode "C,"

$$\text{for } x=0 \quad l = \frac{s}{I}\sqrt{\frac{2\lambda(\vartheta_0 - \vartheta_1)}{\rho_0}}, \tag{6C}$$

$$\text{for } x=l \quad l = \frac{s}{I}\sqrt{\frac{2\lambda(\vartheta_1 - \vartheta_0)}{\rho_0}}. \tag{7C}$$

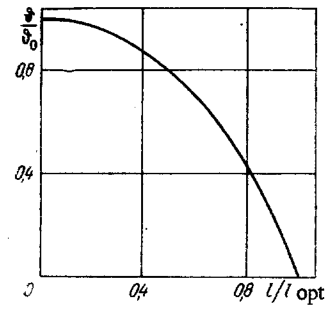

Fig. 1. Temperature distribution over length of lead.

The expression (6C) is meaningful when $\vartheta_0 > \vartheta_1$, and (7C) is meaningful when $\vartheta_1 > \vartheta_0$ (i.e., in this mode it is possible to compensate the heat flux at the hotter end of the conductor).

We note that (6C) and (7C) satisfy McFee's conditions for optimization of cryogenic current inputs [2].

Thus we may use (6), (7), which relate the conductor geometry (s, l, F) to the conductor material (ρ_0, λ, β) and to the operating regime (ϑ_0, ϑ_1, I, α) in order to obtain compensation of the heat flow along current-carrying leads.

Let us examine a sample calculation and an experimental check of our proposed relationships. We consider a katharometer cell for investigation of gas thermal conductivity at low temperatures; it is a cylinder 8 mm in diameter filled with gaseous nitrogen. A type D9D semiconductor diode is used as the sensor. Both calculation and experiment showed a commercially produced sensor with 0.5-mm diameter platinum-wire leads to be unsuitable, since most of the heat flux applied to the sensor is dissipated not from the sensor surface but by conduction along the leads. Analyzing several arrangements with leads of different materials and various geometries, we chose one: the leads to the sensor were made of 0.5 mm diameter nickel wire. The following values were used in calculations for these leads: cell wall temperature T_W = 90°K, sensor surface temperature 120°K; $\vartheta_0 = 30°$; $\vartheta_1 = 0$; $\beta = 5 \cdot 10^{-2}$ 1/deg; $\rho_0 = 1.02 \cdot 10^{-8}$ $\Omega \cdot$m; $\lambda = 112$ W/m·deg; $F = 1.57 \cdot 10^{-4}$ m; $s = 1.96 \cdot 10^{-9}$ m^2; I = 0.2A; $\alpha = 100$ W/m·deg.

Let us determine the mode in which the leads to a sensor having the above parameters operate:

$$\sqrt{\frac{\alpha F s}{\beta \rho_0}} = \sqrt{\frac{100 \cdot 1{,}57 \cdot 10^{-4} \cdot 1{,}96 \cdot 10^{-9}}{5 \cdot 10^{-2} \cdot 1{,}02 \cdot 10^{-8}}} = 0{,}247A.$$

Since $I < \sqrt{(\alpha Fs/\beta \rho_0)}$, the leads operate in mode "B." We find c/k^2:

$$c = \frac{I^2 \rho_0}{\lambda s^2} = \frac{0{,}04 \cdot 1{,}02 \cdot 10^{-8}}{112 \cdot 3{,}82 \cdot 10^{-18}} \simeq 9{,}8 \cdot 10^5 \ (\text{deg/m}^2),$$

$$k^2 = \frac{\alpha F}{\lambda s} - \frac{I^2 \rho_0}{\lambda s^2} \beta = \frac{100 \cdot 1{,}57 \cdot 10^{-4}}{112 \cdot 1{,}96 \cdot 10^{-9}} - 9{,}8 \cdot 10^5 \cdot 5 \cdot 10^{-2} = 2{,}35 \cdot 10^{-4} (1/м^2),$$

$$k \simeq 1{,}53 \cdot 10^2 \ (1/м),$$

$$\frac{c}{k^2} = \frac{9{,}8 \cdot 10^5}{2{,}35 \cdot 10^4} \simeq 42 \ (\text{deg}).$$

Since $c/k^2 > \vartheta_0 > \vartheta_1$, the heat influx can be compensated at the point $l = 0$. The optimal length of the lead is

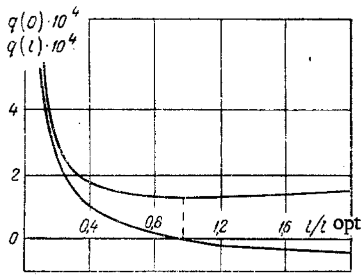

Fig. 2. Heat influx (q, W) at cold and hot ends of the lead as a function of the length chosen for it.

$$l = \frac{1}{k}\ \text{Arch}\frac{\vartheta_1 - \frac{c}{k^2}}{\vartheta_0 - \frac{c}{k^2}} = \frac{1}{1{,}53\cdot 10^2}\ \text{Arch}\frac{0-42}{30\cdot 42} \simeq 12{,}6\cdot 10^{-3}\ \text{(m)}.$$

Thus when the commercial-model leads are replaced by 12.6-mm long nickel leads we create the conditions for compensation of the heat influx from the sensor to the walls of the katharometer cell. Tests of a sensor with such leads confirmed that it is capable of good operation. In practice the heat flowing in along the leads did not exceed 5% of the total power dissipated from the sensor surface.

The temperature distribution along the length of the lead (Fig. 1) and the dependence of the heat influxes along the lead on its length (Fig. 2) are given for the example considered.

NOTATION

I	is the electric current;
α	is the heat transfer coefficient between the conductor surface and the ambient medium;
x	is the coordinate for the running length of the conductor;
l, s, and F	are the length, cross-section area, and perimeter of the conductor;
λ	is the thermal conductivity of the conductor material;
ρ_0	is the electrical resistivity of the conductor at the ambient temperature T_W;
β	is the relative temperature coefficient of resistance;
ϑ_0 and ϑ_1	are the temperatures of the conductor ends with respect to the ambient temperature T_W.

LITERATURE CITED

1. P. A. Korotkov et al., Thermal Flowmeters [in Russian], Mashinostroenie, Leningrad (1969).
2. McFee, Rev. Sci. Instr., 30, No. 2, 98-105 (1959).